NMSU MATH PROBLEM OF THE WEEK

Solution to Problem 5

Spring 2026

Problem 5

At an NMSU alumni get together, there are 30 people in attendance. Due to the short duration of the event, each person knows between 1 and 9 other people in the room. Prove that there must be at least four people in the room who know the exact same number of other people. *(Assume that if person A knows person B, then person B knows person A).*

Solution. We use the generalized pigeonhole principle to solve this problem. Recall that the ceiling function is defined by

$$\lceil x \rceil = \text{ the least integer greater than or equal to } x.$$

By the generalized pigeonhole principle, if n pigeons are placed in k boxes, at least one box must contain $\lceil \frac{n}{k} \rceil$ or more pigeons. In our case, we have 9 pigeonholes (each person knows 1 to 9 people in the room) and 30 pigeons (total of 30 people in attendance). Therefore, at least one pigeonhole must contain $\lceil \frac{30}{9} \rceil = 4$ pigeons. Thus, there must be at least 4 people in the room who know the exact same number of other people. $\square$